Problem Set

1. Do the following parts (a) to (c).

a. Find and solve the Euler equation for: subject to  Do not use the boundary conditions to solve for the constants in the solution.

b. Find and solve the Euler equation for: subject to  Do not use the boundary conditions to solve for the constants in the solution.

c. Show that the solution to the problem subject to  is straight line if  or 

2. A consumer has intertemporal preferences represented by  where  is consumption,  if  and  and  if  The consumer faces the within-period budget constraint  where  is capital and  is the interest rate. Capital and consumption are constrained to be positive,  and  Use a calculus of variations approach to find the optimal path of consumption.

3. Use a control theory approach to solve the problems

a.  subject to  and 

b.  subject to  and 

4. A representative consumer’s intertemporal utility is given by  where  the path of consumption,  is the path of labor and  is a well-behaved utility function. The consumer’s within period budget constraint is given by   where  is the path of the capital stock,  is the interest rate,  are holdings of government bonds,  is a lump-sum tax and  is a well-behaved constant-returns-to-scale production function. The consumer’s initial holdings of bonds and capital are given.

a. Write down the Hamiltonian and find the optimality equations. Demonstrate that the marginal produce of capital is equal to the interest rate.

b. Write down the terminal equations and provide and intuitive explanation. (Two or three sentences will suffice.)

c. The government’s budget constraint is  where  is government spending. Demonstrate that 

5. Consider a model of a small open economy with sticky prices:

 

where  and  are output, the exchange rate, the price level, the money supply and the interest rate, respectively. Variables are in logarithmic form [and hence can be negative].

a. Suppose that the initial price is given and that *m =* 10. Solve for  and  requiring that the economy be on a saddle path.

b. Suppose that the economy is at a steady state with *m =* 10.There is an unexpected jump of the money supply to *m* = 20. It is expected that the money supply will remain constant at this level. Analyze graphically the path to the new steady state.

6. Consider an overlapping-generations model with the equilibrium condition

 

where *y* is constant output and  is time-*t* real balances. The function *h* is the inverse of the marginal rate of substitution.

a. What are the possible equilibria when consumers have preferences over consumption when young and consumption when old that are represented by 

b. Can there be non-stationary equilibria when consumers have preferences over consumption when young and consumption when old that are represented by  A graphical argument will suffice.